INDE

0567
0685
0793
1133
1178
1190
2358
6110
7039
7040

UNCLASSIFIED

Distribution

Prepared under

Contract No. Nonr-3815(00)(X)
Task NR 062-297

Supported jointly by

Department of Defense
Atomic Energy Commission
National Aeronautics and
Space Administration

Administered by

S. Doroff
Fluid Dynamics Branch
Office of Naval Research
Department of the Navy

THE DYNAMIC BEHAVIOR OF A FOIL IN THE
PRESENCE OF A LUBRICATING FILM
by
A. Eshel and M. Wildmann
RR 66-29   September 1966

Prepared by *A. Eshel*

and *M. Wild*

Approved by *N. Tross*

AMPEX CORPORATION

RESEARCH AND ENGINEERING PUBLICATION

UC-271

ABSTRACT

Equations for the oscillations of a foil over
a lubricating fluid film are derived and are simplified by a
small parameter expansion. A few particular cases are dis-
cussed, and a linearized solution is obtained for the case
of a massless, perfectly flexible foil moving at a speed U
over an incompressible film. The solution reveals the
phenomenon that small disturbances in the film thickness,
as well as symmetrical large disturbances, propagate at a
speed U/2.

CONTENTS

LIST OF FIGURES

NOMENCLATURE

a   Half square wavelet width

$\bar{a}$ Normalized half square wavelet width $\equiv a/\tau^{1/4}$

a_n Foil acceleration component normal to local foil direction

a_s Foil acceleration component along local foil direction

A   Height of square wavelet

C   Compressibility parameter $\equiv \dfrac{p_a\,h_o}{T_o}\,\epsilon^{-2/3}$

d   Local foil thickness

d_o Unstretched foil thickness

$\bar{d}$ Dimensionless foil thickness d/d_o

D   Flexural rigidity of tape per unit width

E   Modulus of elasticity

f   Small perturbation $\equiv H-1$

f_o Initial small disturbance

F   Fourier transform of f

F_o Transform of initial disturbance

h   Clearance

h_o Asymptotic clearance

H — Dimensionless clearance $\equiv h/h_o$

j,k,ℓ,m,n — Exponential measures

M — Bending moment per unit width

$\bar{M}$ — Normalized moment

M_p — Mass parameter $\equiv \dfrac{\sigma_o U_o}{12\mu} \epsilon$

p — Pressure under foil

p_a — Ambient pressure

Q — Shear force per unit width

R — Local radius of curvature of foil

s — Distance along tape; Fourier transform parameter

$\bar{s}$ — Normalized distance along tape $\equiv \dfrac{s}{h_o} \epsilon^{1/3}$

S_p — Stiffness parameter $\equiv \dfrac{D \epsilon^{2/3}}{T_o h_o^2}$

T — Tension per unit width of foil

T_o — Tension per unit width of foil at $s = \infty$

$\bar{T}$ — Normalized tension $\equiv T/T_o$

T_p — Tension parameter $\equiv \dfrac{T_o}{E d_o}$

t — Time

U — Foil velocity component normal to local foil direction

U_o — Foil velocity at $s = \infty$

$\bar{U}$ — Dimensionless velocity component in foil direction $\equiv \dfrac{U}{U_o}$

W — Foil velocity component normal to local foil direction

$\overline{W}$ Normalized velocity component normal to foil direction $\equiv \dfrac{W}{U_o} \epsilon^{-1/3}$

x   Longitudinal coordinate

ϵ Dimensionless parameter $\dfrac{6\mu\, U_o}{T_o}$

χ Dimensionless curvature

λ $\equiv 2\pi\,/$(wavelength of Fourier component)

$\overline{\lambda}$ $\lambda\, \tau^{1/4}$

μ Viscosity

ν Poisson ratio

π Dimensionless pressure $\dfrac{p - p_a}{T_o / h_o} \epsilon^{-2/3}$

σ Foil mass per unit area

σ_o Unstretched foil mass per unit area

$\overline{\sigma}$ Dimensionless foil mass per unit area

τ Fluid shear traction on foil; dimensionless time $\dfrac{t\, U_o}{2\, h_o} \epsilon^{-1/3}$

θ Slope

ξ Normalized longitudinal coordinate $\equiv \dfrac{x}{h_o} \epsilon^{1/3}$

1.0   INTRODUCTION

The dynamics of a flexible foil serving as a boundary for the flow of a thin fluid film is a problem of considerable interest and application. The use and manufacture of magnetic tapes, paper, and plastic foils are some of the fields in which such applications arise.

Previous studies of this elasto-hydrodynamic problem were confined to steady state. It is the purpose of this paper to formulate and obtain a solution for a simple model involving transient phenomena in order to gain understanding and make it possible to interpret and devise techniques for the solution of more complex configurations which arise in practice.

2.0   DESCRIPTION OF THE MODEL

Consider an infinitely wide foil stretched between two far apart guides (Fig. 1) and moving at a very small distance, h_o, from a solid surface. Neglecting gravity, the equilibrium configuration of the foil may be found to be a straight line connecting the supports. At time $t = 0$, a disturbance in the shape of the foil is introduced, and the problem is to find its subsequent history.

The coordinates used in the mathematical description of the problem are t (time) and x (stationary position coordinate measured along the solid surface). Vectorial resolutions into components are made along the instantaneous positions of the foil (subscript s) and normal to it (subscript n). Some of the equations are more conveniently derived using the foil length s as a coordinate, rather than the stationary coordinate x.
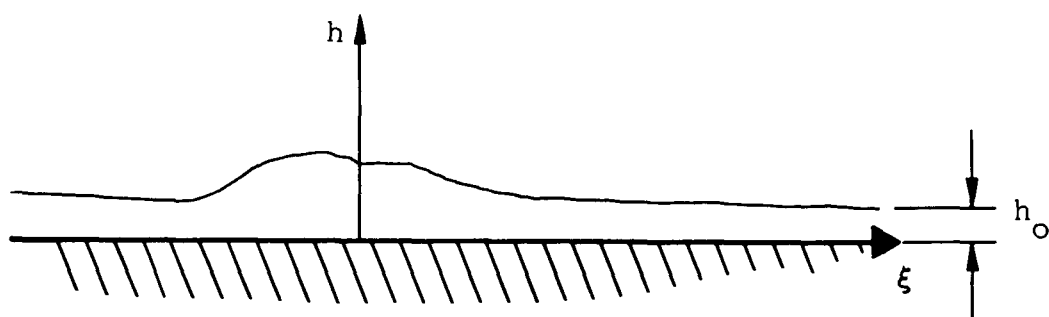
Fig. 1   Schematic View of Problem Under Consideration

The conversion between the two coordinates is

$$\left(\frac{\partial s}{\partial x}\right)_t = \left[1 + \left(\frac{\partial h}{\partial x}\right)_t^2\right]^{1/2} \tag{1}$$

where h is the clearance.

3.0 ASSUMPTIONS

(1) The flow is assumed to be planar.

(2) The fluid velocity profile is parabolic, and the fluid inertia is neglected.

(3) For consistency with assumption (1), the approximation is made that the lateral strain in the foil is zero.

(4) The stresses normal to the foil and the effect of the transverse shears on the deformation are neglected in analogy with the theory of plates.

(5) The rotational inertia of the foil is neglected.

4.0 BASIC EQUATIONS

Using assumptions (1) and (2), the fluid continuity is expressed by the Reynolds equation,

$$\frac{\partial}{\partial x}\left(h^3 p \frac{\partial p}{\partial x}\right) = 6\mu \frac{\partial(u p h)}{\partial x} + 12\mu \frac{\partial(p h)}{\partial t} \tag{2a}$$

where p, u are the pressure and the tangential component of the tape speed, respectively, which are functions of position and time. Based on

the same approximation is the expression for the fluid shear tractions on the foil

$$\mathcal{T} = -\frac{h}{2}\frac{\partial p}{\partial x} - \mu\frac{U}{h} \tag{2b}$$

The three dynamic equations on an element of foil are:

$$p - p_a - \frac{T}{R} + \frac{dQ}{ds} = \sigma a_n \tag{2c}$$

$$\frac{dT}{ds} + \frac{Q}{R} + \mathcal{T} = \sigma a_s \tag{2d}$$

$$\frac{dM}{ds} - Q = 0 \tag{2e}$$

The right hand side of the last equation vanishes due to assumption (5). In the above equations $T, \sigma, Q, M, R, a_n, a_s$ denote tension per unit width, foil mass per unit area, normal shear force, bending moment, radius of curvature, normal and tangential acceleration components, respectively. The aforementioned variables are functions of position and time. With assumptions (3) and (4), the stress-strain relations result in the equations,

$$d = d_o - \frac{\nu(1+\nu^2)}{E}T \tag{2f}$$

$$\frac{1-\nu^2}{E}\frac{\partial\left(\frac{T}{d}\right)}{\partial t} = \frac{\partial U}{\partial s} \tag{2g}$$

$$M = \frac{D}{R} \tag{2h}$$

where d is the local tape thickness which is a function of position and time, and D is the flexural rigidity per unit width:

$$D \equiv \frac{E d^3}{12(1 - \nu^2)}$$

The continuity equation for the tape is

$$\frac{\partial}{\partial x}\left(\sigma \cdot d \left[U \cos \theta - W \sin \theta \right]\right) = -\frac{\partial}{\partial t}\left(d \cdot \sigma\right) \tag{2j}$$

where $\theta = \theta(x, t)$ is the slope of the foil and $W(x, t)$ is the normal component of the tape speed. The acceleration components of an element of foil are*:

$$a_t = \frac{DU}{Dt} - \frac{D\theta}{Dt} W \tag{2l}$$

$$a_n = \frac{DW}{Dt} + \frac{D\theta}{Dt} U \tag{2m}$$

---

*The operator Df/Dt denotes the convective derivative, i.e., the rate of change of the property f following a material point of the tape. In (x, t) and (s, t) expressions, this becomes, respectively:

$$\frac{Df}{Dt} \equiv \left(\frac{\partial f}{\partial x}\right)_t \left(U \cos \theta - W \sin \theta\right) + \left(\frac{\partial f}{\partial t}\right)_x$$

$$\equiv \left(\frac{\partial f}{\partial s}\right)_t \left(\frac{Ds}{Dt}\right) + \left(\frac{\partial f}{\partial t}\right)_s$$

Finally, the following kinematical relations are needed to complete the formulation:

$$\frac{1}{R} = - \frac{\frac{\partial^2 h}{\partial x^2}}{\left[1 + \left(\frac{\partial h}{\partial x}\right)^2\right]^{3/2}} \tag{2n}$$

$$\frac{Dh}{Dt} = U \sin\theta + W \cos\theta \tag{2o}$$

$$\left(\frac{\partial h}{\partial x}\right)_t = tg\,\theta \tag{2p}$$

5.0   ANALYSIS

In the following, the equations will be nondimensionalized and then simplified by means of an asymptotic analysis. The analysis revolves around the dimensionless group $\epsilon = \frac{6\mu U_0}{T_0}$ which in view of past experience[1], is chosen as a perturbation parameter. The dependent variables will be normalized with respect to their characteristic magnitude so that the new dimensionless quantity will be of order unity. The independent variables will be normalized with respect to their characteristic interval of change so that differentiation with respect to the resulting dimensionless variable will not change the order of magnitude of the differentiated dimensionless variable. Those characteristic dimensions which are unknown a priori are formed with the aid of unknown exponents which are to be determined later.

Thus,

$$\bar{U} \equiv \frac{U}{U_o} \tag{3a}$$

$$\bar{d} \equiv \frac{d}{d_o} \tag{3b}$$

$$\bar{G} \equiv \frac{G}{G_o} \tag{3c}$$

$$H \equiv \frac{h}{h_o} \tag{3d}$$

$$\bar{T} \equiv \frac{T}{T_o} \tag{3e}$$

$$\pi \equiv \frac{p - p_a}{\frac{T_o}{h_o} \epsilon^k} \tag{3f}$$

$$\chi \equiv \frac{h_o / \epsilon^n}{R} \tag{3g}$$

$$\bar{W} \equiv \frac{W}{U_o \epsilon^\ell} \tag{3h}$$

$$\tau \equiv \frac{t}{\frac{2h_o}{v_o}/\epsilon^j} \tag{3i}$$

$$\bar{s} \equiv \frac{s}{h_o/\epsilon^m} \tag{3j}$$

$$\xi \equiv \frac{x}{h_o/\epsilon^m} \tag{3k}$$

The new variables Eq. (3) are substituted in Eqs. (2). In addition, Eq. (2h) is substituted in Eq. (2e) and Eqs. (2b), (2e), (2 l), (2m) into Eqs. (2c), (2d). Finally, Eq. (2p) is expanded into a trigonometric series as follows:

$$\theta = arc\, tg\left(\epsilon^m \frac{\partial H}{\partial \xi}\right) = \epsilon^m \frac{\partial H}{\partial \xi} - \frac{\epsilon^{3m}\left(\frac{\partial H}{\partial \xi}\right)^3}{3} + \cdots \tag{4}$$

and the transformation

$$\left(\frac{\partial}{\partial \bar{s}}\right)_t \equiv \left[1 - \frac{1}{2}\frac{\partial H}{\partial \xi}\epsilon^{2m}\cdots\right]\frac{\partial}{\partial \xi} \tag{5}$$

is applied. With these changes, Eqs. (2) become:

$$\epsilon^{2m+2k}\frac{\partial}{\partial\xi}\left(H^3\left[\pi+\frac{p_a h_0}{T_0\epsilon^k}\right]\frac{\partial\pi}{\partial\xi}\right)=\epsilon^{1+m+k}\frac{\partial\left(\bar{U}\left[\pi+\frac{p_a h_0}{T_0\epsilon^k}\right]H\right)}{\partial\xi}+\epsilon^{1+k+j}\frac{\partial\left(\left[\pi+\frac{p_a h_0}{T_0\epsilon^k}\right]H\right)}{\partial\tau} \tag{6a}$$

$$\frac{\partial\bar{T}}{\partial\xi}\left[1-\tfrac{1}{2}\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right]+\tfrac{1}{2}\frac{D\epsilon^{2n}}{T_0 h_0^2}\frac{\partial^2\chi}{\partial\xi}\left[1-\tfrac{1}{2}\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right]-\frac{\epsilon^k}{2}H\frac{\partial\pi}{\partial\xi}-\frac{\epsilon^{l-m}}{6}\frac{\bar{U}}{H}=\frac{\sigma_0 U_0}{12\mu}\epsilon^{1+j-m}\bar{d}\left[\frac{D\bar{U}}{Dt}-\frac{D}{Dt}\left(\frac{\partial H}{\partial\xi}\right)\bar{W}\epsilon^{l+m}\right] \tag{6b}$$

$$\pi-\epsilon^{n-k}\bar{T}\chi+\frac{D\epsilon^{n+2m-k}}{T_0 h_0^2}\frac{\partial^2\chi}{\partial\xi^2}\left[1+\cdots\right]=\frac{\sigma_0 U_0}{12\mu}\epsilon^{1+j-k}\bar{d}\left(\frac{D\bar{W}}{D\tau}\epsilon^l+\frac{D}{D\tau}\left(\frac{\partial H}{\partial\xi}\right)\bar{U}\epsilon^m+\cdots\right) \tag{6c}$$

$$\bar{d}=1-\nu(1+\nu^2)\frac{T_0}{Ed_0}\bar{T} \tag{6d}$$

$$\frac{1-\nu^2}{2}\frac{T_0}{Ed_0}\frac{\partial\left(\frac{\bar{T}}{\bar{d}}\right)}{\partial\tau}=\epsilon^{m-j}\frac{\partial\bar{U}}{\partial\xi}\left[1-\tfrac{1}{2}\frac{\partial H}{\partial\xi}\epsilon^{2m}\right] \tag{6e}$$

$$2\epsilon^{m-j}\frac{\partial}{\partial\xi}\left(\bar{\sigma}\,\bar{d}\left[\bar{U}\left(1-\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right)\right]-\bar{W}\epsilon^{l+m}\frac{\partial H}{\partial\xi}\cdots\right)=-\frac{\partial(\bar{\sigma}\bar{d})}{\partial\tau} \tag{6f}$$

$$\chi=-\epsilon^{2m-n}\frac{\partial^2 H}{\partial\xi^2}\left[1-\tfrac{3}{2}\left(\frac{\partial H}{\partial\xi}\right)^2\epsilon^{2m}\cdots\right] \tag{6g}$$

$$\epsilon^j\frac{DH}{D\tau}=2\left(\bar{U}\frac{\partial H}{\partial\xi}\epsilon^m+\bar{W}\epsilon^l\cdots\right) \tag{6h}$$

The already mentioned perturbation process with $\epsilon \to 0$ will be considered now. In the physical problem which is of interest here, namely, the time dependent problem with interaction between the fluid-mechanical effects and the elastic effects in the foil, all the terms of Eq. (6a) are to be of the same order, hence:

$$2m + 2k = 1 + m + k \tag{7a}$$

$$2m + 2k = 1 + k + j \tag{7b}$$

Since it is desired that x and $\dfrac{\partial^2 H}{\partial \xi^2}$ will be of order unity it follows from Eq. (6j) that it must be true that

$$2m - n = 0 \tag{7c}$$

By similar reasons it is concluded from Eq. (6c) that

$$n - k = 0 \tag{7d}$$

As a result of Eq. (6h):

$$\ell = j$$
$$m = j \tag{7e}$$

(Note that the last condition is redundant). The solution of Equations (7) is:

$$m = 1/3 \tag{8a}$$
$$n = 2/3 \tag{8b}$$
$$k = 2/3 \tag{8c}$$
$$j = 1/3 \tag{8d}$$
$$1 = 1/3 \tag{8e}$$

It is further concluded from Eq. (6c) that foil stiffness is important when the stiffness parameter

$$S_p \equiv \frac{D \, \epsilon^{2/3}}{T_o \, h_o^2} \sim O(1) \tag{9}$$

and foil mass is important when the mass parameter

$$M_p \equiv \frac{\sigma_o \, U_o}{12 \, \mu} \epsilon \sim O(1) \tag{10}$$

Variations in tension, speed and thickness are significant when the extensibility parameter

$$T_p \equiv \frac{T_o}{E \, d_o} \sim O(1) \tag{11}$$

and the effect of fluid compressibility is appreciable when the compressibility parameter

$$\frac{1}{C_p} \equiv \frac{T_o \, \epsilon^{2/3}}{p_a \, h_o} \sim O(1) \tag{12}$$

The functions H, π, $\bar{T}$, $\bar{d}$, $\bar{U}$, $\bar{W}$, x, $\bar{\delta}$ are now expanded in a series of the form:

$$P = P_o + P_1 \, \epsilon^{2/3} + \ldots \tag{13}$$

where P stands for any one of the above property functions. Substitution of the expansion Eq. (13), into the equations (6) collecting terms of equal powers of $\epsilon^{2/3}$, the zeroth approximation with the omission of the subscript zero becomes:*

---

*If consistent higher order approximation equations are desired it is necessary to use more accurate fluid mechanical equations and at a certain stage, also, more accurate elastic equations.

$$\frac{\partial}{\partial \xi}\left(H^3[\pi + C_p]\frac{\partial \pi}{\partial \xi}\right) = \frac{\partial(\bar{U}[\pi + C_p]H)}{\partial \xi} + \frac{\partial([\pi + C_p]H)}{\partial \tau} \tag{14a}$$

$$\frac{\partial \bar{T}}{\partial \xi} = M_p \bar{\sigma}\frac{D\bar{U}}{D\tau} \tag{14b}$$

$$\pi - T\chi + S_p\frac{\partial^2 \chi}{\partial \xi^2} = M_p \bar{\sigma}\left[\frac{D\bar{W}}{D\tau} + \frac{D}{D\tau}\left(\frac{\partial H}{\partial \xi}\right)\right] \tag{14c}$$

$$\bar{d} = 1 - \nu'(1 + \nu^2)T_p \bar{T} \tag{14d}$$

$$\frac{1 - \nu^2}{2}T_p\frac{\partial\left(\frac{\bar{T}}{\bar{d}}\right)}{\partial \tau} = \frac{\partial \bar{U}}{\partial \xi} \tag{14e}$$

$$\frac{\partial}{\partial \xi}\left(\bar{\sigma}\bar{d}\bar{U}\right) = -\frac{1}{2}\frac{\partial(\bar{\sigma}\bar{d})}{\partial \tau} \tag{14f}$$

$$\chi = -\frac{\partial^2 H}{\partial \xi^2} \tag{14g}$$

$$\frac{DH}{D\tau} = 2\left(\bar{U}\frac{\partial H}{\partial \xi} + \bar{W}\right) \tag{14h}$$

where the operator $\dfrac{D}{D\tau}$ to the zeroth approximation is:

$$\frac{D}{D\tau} = 2\bar{U}\frac{\partial}{\partial \xi} + \frac{\partial}{\partial \tau}$$

6.0 SPECIAL CASES

6.1 Inexstensible Foil

In this case $T_p \sim 0(\epsilon^{2/3})$. If at $\xi = -\infty$, $\bar{T} = 1$, $\bar{U} = 1$ and if initially $\bar{\sigma}(\xi) \equiv 1$ then $\bar{d} = 1$. $\bar{\sigma} = 1$ at all times and

$$\frac{\partial}{\partial \xi}\left(H^3[C_p + \pi]\frac{\partial \pi}{\partial \xi}\right) = \frac{\partial}{\partial \xi}\left([C_p + \pi]H\right) + \frac{\partial}{\partial \tau}\left([C_p + \pi]H\right) \tag{15a}$$

$$\pi + \frac{\partial^2 H}{\partial \xi^2} - S_p\frac{\partial^4 H}{\partial \xi^4} = 2 M_p\left(\frac{\partial^2 H}{\partial \xi^2} + \frac{\partial^2 H}{\partial \tau \partial \xi} + \frac{1}{4}\frac{\partial^2 H}{\partial \tau^2}\right) \tag{15b}$$

It is seen that in this case the effect of the foil mass appears only transversely. If extensionality is not negligible then the foil mass is of equal significance or insignificance transversely and longitudinally.

6.2 Massless Foil

In this case $M_p \sim 0(\epsilon^{2/3})$. If at $\xi = -\infty$, $\bar{T} = 1$, then $\bar{U} = 1$, $\bar{\sigma} = 1$, $\bar{d} = 1 - \nu(1 + \nu^2) T_p$ at all times and the equations reduce to

$$\frac{\partial}{\partial \xi}\left(H^3[C_p + \pi]\frac{\partial \pi}{\partial \xi}\right) = \frac{\partial}{\partial \xi}\left([C_p + \pi]H\right) + \frac{\partial}{\partial \tau}\left([C_p + \pi]H\right) \tag{16a}$$

$$\pi + \frac{\partial^2 H}{\partial \xi^2} - S_p\frac{\partial^4 H}{\partial \xi^4} = 0 \tag{16b}$$

6.3   Solid Moving at U_o; No Axial Bulk Motion of Foil

Two cases must be distinguished here:

Case a: Large disturbances causing longitudinal foil speeds of order U_o are considered. In this case the speed in the Quette terms in the Reynolds equation and in the fluid shear expression has to be modified to include the sum of U_o and U_{foil}. In dimensionless form, to the zeroth approximation, the only modification in Eqs. (14) will be the replacement of $\bar{U}$ by $1 + \bar{U}$ in Eq. (14a). (The effect on the fluid shear is, anyway, of order $\epsilon^{2/3}$.)

Case b: Only longitudinal disturbances which do not affect the fluid mechanics will be considered. Thus, foil speeds of order $U_o \, \epsilon^{2/3}$ or less are considered. To the zeroth order approximation:

$$\frac{D}{D\tau} \equiv \frac{\partial}{\partial\tau} + O\left(\epsilon^{2/3}\right)$$

$$\frac{\partial H}{\partial\tau} = 2\bar{W}$$

$$\frac{\partial \bar{T}}{\partial \xi} = O\left(\epsilon^{2/3}\right)$$

The basic equations become to the zeroth order approximation

$$\frac{\partial}{\partial\xi}\left(H^3[\pi + C_p]\frac{\partial\pi}{\partial\xi}\right) = \frac{\partial([\pi + C_p]H)}{\partial\xi} + \frac{\partial([\pi + C_p]H)}{\partial\tau} \tag{17a}$$

$$\pi + \frac{\partial^2 H}{\partial\xi^2} - S_p\frac{\partial^4 H}{\partial\xi^4} = 2 M_p \bar{\sigma}\,\frac{\partial^2 H}{\partial\tau^2} \tag{17b}$$

6.4 Massless, Perfectly Flexible Foil, Incompressible
     Flow

The equations reduce to

$$\frac{\partial}{\partial \xi}\left(H^3 \frac{\partial^3 H}{\partial \xi^3}\right) + \left(\frac{\partial H}{\partial \xi}\right) = -\frac{\partial H}{\partial \tau}$$

(18)

Writing the equation in terms of a coordinate $\xi' = \xi - \tau$ moving at a speed $\frac{d\xi}{d\tau} = 1$, it becomes:

$$H^3 H^{IV} + 3 H^2 H' H''' = -\frac{\partial H}{\partial \tau}$$

It is clear, therefore, that symmetrical disturbances will make $\frac{\partial H}{\partial \tau}$ symmetrical and will stay symmetrical as time goes on. Thus, the axis of symmetry of such disturbances travels at a dimensionless velocity of $\frac{d\xi}{d\tau} = 1$ which corresponds to an actual velocity of $U/2$. It will be shown below that small disturbances of arbitrary shape also propagate at a velocity of $U/2$. This is not necessarily true, however, for large non-symmetrical disturbances. For example, it can be shown that the "nodal" point, $H = 1$, of an antisymmetrical large disturbance moves at a speed different from $U/2$.

If the disturbance is small relative to the magnitude of the clearance, the equation may be linearized as follows:

$$H = 1 + f(\xi, \tau)$$

(19)

$$\frac{\partial^4 f}{\partial \xi^4} + \frac{\partial f}{\partial \xi} = -\frac{\partial f}{\partial \tau}$$

(20)

with $f = f_o(\xi)$ at $\tau = 0$.

The solution is obtained by taking the Fourier transform:

$$F(s,\tau) \equiv \int_{-\infty}^{\infty} f(\xi,\tau)\, e^{-2\pi\xi\, s i}\, d\xi \tag{21}$$

The transformed differential equation is

$$\frac{dF}{d\tau} + \left(2\pi s i + 16\pi^4 s^4\right) F = 0 \tag{22}$$

and its solution

$$F(s,\tau) = F_o(s)\, e^{-\left(2\pi s i + 16\pi^4 s^4\right)\tau} \tag{23}$$

where $F_o(s)$ is the transformed initial condition. Inversion is made by two successive uses of the convolution theorem and the use of a new variable $\lambda = 2\pi s$. It is found that

$$H(\xi,\tau) = 1 + \frac{1}{\pi} \int_{-\infty}^{\infty} \int_{0}^{\infty} f_o(\xi')\, e^{-\lambda^4 \tau} \cos \lambda(\xi - \xi' - \tau)\, d\lambda\, d\xi' \tag{24}$$

Eq.(24) describes wave motion in a dissipative nondispersive medium. Each Fourier component of the disturbance decays at a rate which is proportional to $e^{-\lambda^4 \tau}$ and travels at a constant speed of $U_o/2$. A particular example is given in the appendix.

The above result has been obtained for a case with an infinite distance in terms of ξ between the disturbance and the boundary. Since ξ is a stretched coordinate, this result applies in practice, to cases with finite length between the supports, as long as the propagation of the disturbance has not yet reached within an order of $h_o / \epsilon^{1/3}$ from the boundary. Furthermore, this result applies to the central region of self-acting foil bearing as long as the disturbance is far enough from the inlet and exit regions.

APPENDIX

In this section a specific example is considered in order to enhance the physical understanding of the phenomenon. Imagine an initial disturbance in the form of a square wavelet

$$f_o(\xi) = A \qquad\qquad -a < \xi < a$$

$$f_o(\xi) = 0 \qquad\qquad \xi < -a \; , \quad a < \xi$$

Let the point $\xi = \tau$ be followed. Putting $\lambda\tau^{1/4} = \bar{\lambda}$ and $\dfrac{a}{\tau^{1/4}} = \bar{a}$

Eq. (24) becomes

$$H(\tau,\tau) = 1 + \frac{2A}{\pi}\int_o^\infty \frac{e^{-\bar{\lambda}^4} \sin \bar{a}\bar{\lambda}}{\bar{\lambda}}\, d\bar{\lambda}$$

This integral is numerically evaluated[*] and the result is depicted in Fig. A1. When $\bar{a}$ is small (large τ) the integral may be approximated by:

$$\frac{H(\tau,\tau)-1}{2A/\pi} \cong \bar{a}\int_o^\infty e^{-\bar{\lambda}^4}d\bar{\lambda} - \frac{\bar{a}^3}{3!}\int_o^\infty \bar{\lambda}^2 e^{-\bar{\lambda}^4}d\bar{\lambda} + \dots$$

$$= 0.906402\left(\frac{a}{\tau^{1/4}}\right) - 0.051059\left(\frac{a}{\tau^{1/4}}\right)^3 + \dots$$

The asymptotic approximation is represented by a broken line in Fig. A1.

---

[*]The apparent singularity in the integral is overcome by using the relation

$$\frac{H(\tau,\tau)-1}{2A/\pi} = \bar{a}\cdot\delta - \frac{\bar{a}^3\delta^3}{3\cdot 3!} + \int_\delta^\infty \frac{e^{-\bar{\lambda}^4}\sin \bar{a}\bar{\lambda}}{\bar{\lambda}}\,d\bar{\lambda} + O\left([\delta\cdot\bar{a}]^5\right)$$

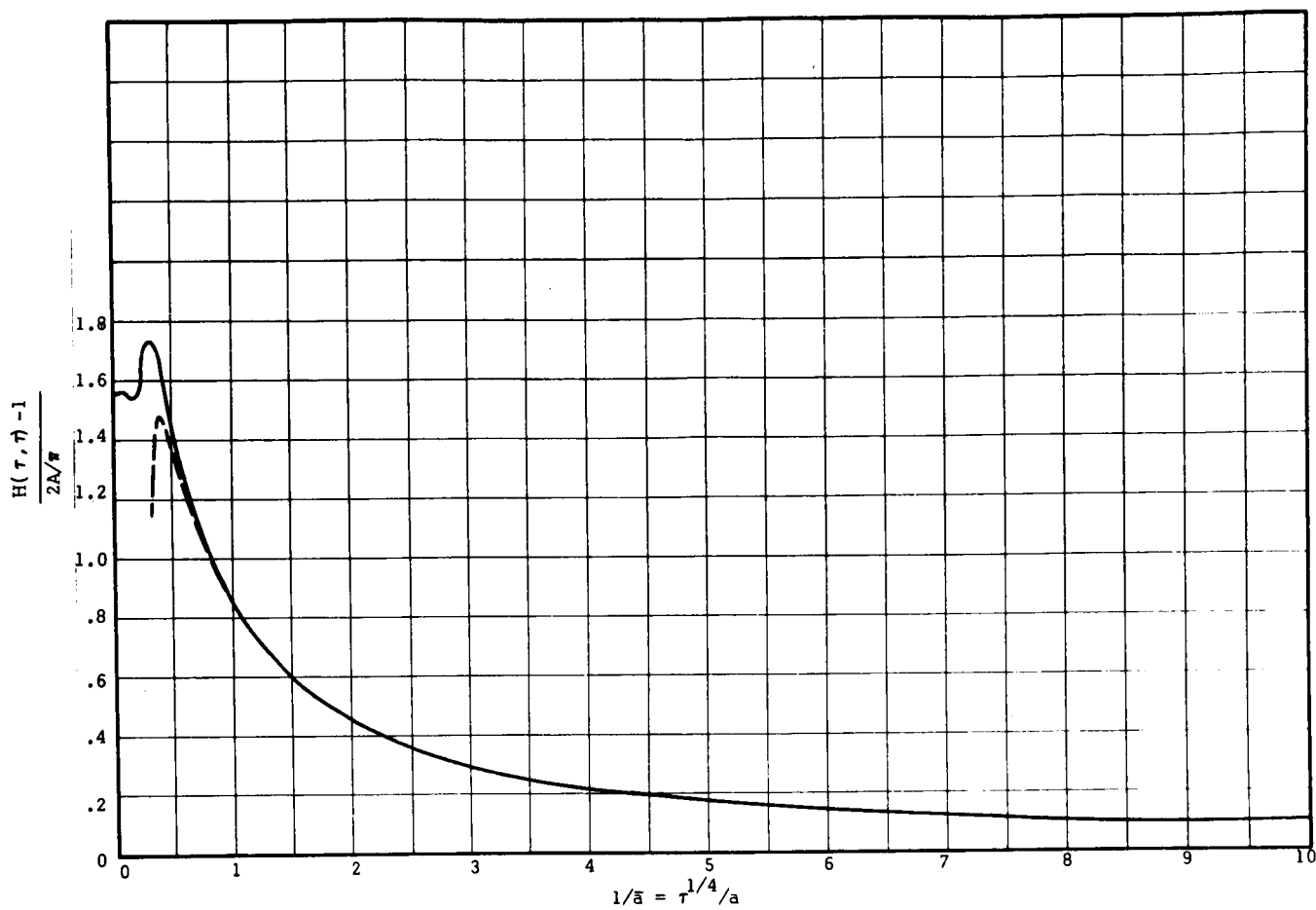where δ is a properly chosen small number.

Fig. A-1   Time History of the Symmetry Point of a Disturbance
in the form of a Square Wavelet

Since the original profile of the disturbance is flat, the symmetry point is initially in a region of uniform clearance and therefore does not change its elevation. The excess fluid spreads out by forming exit regions with their corresponding undulations at the two ends of the wavelet. Thus for a period of time, the symmetry point may be in a region of curvature convex towards the h_o line and is, thus, being pushed away from it until it flattens again. This explains the initial undulations shown in Fig. A1.

REFERENCES

1. Eshel, A. and Elrod, H.G., Jr. "The Theory of the Infinitely Wide, Perfectly Flexible Self Acting Foil Bearing", Journal of Basic Engineering, Trans. ASME, Series D, Vol. 87, 1965, pp. 831-836.

APPROVED DISTRIBUTION LISTS FOR UNCLASSIFIED TECHNICAL REPORTS
ISSUED UNDER
GAS LUBRICATED BEARINGS CONTRACTS

Contract NONR 3815(00)NR 062-297

Chief of Naval Research
Department of the Navy
Washington 25, D. C.
Attn: Code 438 (3)
429
463

Commanding Officer
Office of Naval Research
Branch Office
495 Summer Street
Boston, Massachusetts 02110

Commanding Officer
Office of Naval Research
Branch Office
230 N. Michigan Avenue
Chicago, Illinois 60601

Commanding Officer
Office of Naval Research
Branch Office
207 West 24th Street
New York, New York 10011

Commanding Officer
Office of Naval Research
Branch Office
Box 39
Fleet Post Office
New York, New York 09510 (25)

Commanding Officer
Office of Naval Research
Branch Office
1030 East Green Street
Pasadena 1, California

Commanding Officer
Office of Naval Research
Branch Office
1000 Geary Street
San Francisco 9, California

Office of Naval Research
Resident Representative
University of Pennsylvania
3636 Walnut Street
Philadelphia, Pennsylvania 19104

Commanding General
U.S. Army Materiel Command
Research & Development Directorate
Research Division
Washington, D. C. 20315
Attn: Mr. Norman L. Klein

Chief of Research and Development
Office of Chief of Staff
Department of the Army
Pentagon Building
Washington 25, D. C.

Commanding General
US Army Engineer R&D Laboratories
Fort Belvoir, Virginia
Attn: W. M. Crim, Nuclear Power
Field Office (2)
Technical Documents Center

Commander
Army Rocket & Guided Missile Agency
Redstone Arsenal, Alabama
Attn: Technical Library

ESD - AROD
Box CM, Duke Station
Durham, North Carolina

Chief of Staff
US Air Force
The Pentagon
Washington 25, D. C.
Attn: AFRDR-AS/M

Commander
Air Force Office of Scientific Research
Washington 25, D. C.
Attn: SRHM

Commander
Research and Technology Division
of the Air Force Systems Command
Wright-Patterson AF Base, Ohio
Attn: APFL (Mr. M. A. Sheets) (3)
APIP-1 (B. L. McFadden, Jr.)
AVWE (R. W. McAdory)
FDFM (P. C. Hanlon)

Defense Documentation Center
Cameron Station
Alexandria, Virginia

Chief, Bureau of Ships
Department of the Navy
Washington 25, D. C.
Attn: Code 305 (Dr. J. H. Huth)
342 (E. A. Bukzin)
644 C (R. M. Petros)
1500 (J. A. Gorman)

Chief, Bureau of Naval Weapons
Department of the Navy
Washington 25, D. C.
Attn: Code RAPP-41 (S. M. Collegeman)
RREN-4 (M. R. Walters)

Director
Naval Research Laboratory
Washington, D. C.
Attn: Code 2000
5230 (6)

Special Projects Office
Department of the Navy
Washington 25, D. C.
Attn: Code SP230 (D. Gold)
SP-001 (Dr. J. P. Craven)

Commanding Officer and Director
US Navy Marine Engineering Laboratory
Annapolis, Maryland
Attn: Code 850
Code 852 (Watt V. Smith)

Material Laboratory Library
Building 291, Code 912B
New York Naval Shipyard
Brooklyn 1, New York

Superintendent
US Naval Postgraduate School
Monterrey, California
Attn: Library, Technical Reports
Section

Commanding Officer
US Naval Avionics Facility
Indianapolis 18, Indiana
Attn: J. G. Weir

Mr. Clarence E. Miller, Jr.
Office of Assistant Director
(Army Reactors)
Division of Reactor Development
US Atomic Energy Commission
Washington 25, D. C. (2)

Mr. N. Grossman, Chief
Engineering Development Branch
Reactor Development Division
US Atomic Energy Commission
Washington 25, D. C. (2)

Headquarters Library
US Atomic Energy Commission
Washington 25, D. C.

US Atomic Energy Commission
Oak Ridge Operations Office
P. O. Box E
Oak Ridge, Tennessee 37831
Attn: Charles A. Keller

US Atomic Energy Commission
Portsmouth Area Office
Piketon, Ohio 45661
Attn: Dr. Malone

Chief, Division of Engineering
Maritime Administration
GAO Building
Washington 25, D. C.

Library
Cryogenic Engineering Laboratory
National Bureau of Standards
Boulder, Colorado

Director of Research, Code RR
National Aeronautics and Space
Administration
600 Independence Avenue
Washington, D. C. 20546

Scientific and Technical Information
Facility
P. O. Box 5700
Bethesda, Maryland 20014
Attn: NASA Representative (SAK/DL)
(2)

Mr. Edmund E. Bisson, Chief
Lubrication and Wear Branch
Lewis Research Center
National Aeronautics and Space
 Administration
21000 Brookpark Road
Cleveland, Ohio

Mr. P. H. Broussard, Jr.
Gyro-Stabilizer Branch
Guidance and Control Division
National Aeronautics and Space
 Administration
George C. Marshall Space Flight Center
Huntsville, Alabama   (5)

Mr. H. W. Savage
Oak Ridge National Laboratory
Post Office Box Y
Oak Ridge, Tennessee

Chief, Technical Information Service
 Extension
Post Office Box 62
Oak Ridge, Tennessee
Attn: Melvin S. Day

Aerojet-General Nucleonics
P. O. Box 86
San Ramon, California
Attn: Document Custodian

Aerospace Corporation
P. O. Box 95085
Los Angeles, California   90045
Attn: Library, Reports Acquisitions
      Group

AiResearch Manufacturing Company
P. O. Box 5217
Phoenix, Arizona   85010
Attn: Mrs. J. F. Mackenzie, Librarian

American Institute of Aeronautics
 and Astronautics
Technical Information Service
750 Third Avenue
New York, New York   10017

American Society of Lubrication
 Engineers
838 Busse Highway
Park Ridge, Illinois   60068

Dr. W. T. Sawyer
Department of Mechanical Engineering
Catholic University
Washington, D. C.

Mr. C. W. Snyder, Manager of Engineering
Defense Engineering
Defense Operations Division
Chrysler Corporation
P. O. Box 757
Detroit 31, Michigan   48231

Robert H. Josephson, Manager
Commercial Development
Clevite Corporation
Mechanical Research Division
540 East 105th Street
Cleveland, Ohio   44108

Professor V. Castelli
Department of Mechanical Engineering
Columbia University
New York, New York

Professor H. Elrod
Department of Mechanical Engineering
Columbia University
New York, New York   (2)

Mr. Gerald B. Speen
Division Manager
Conductron
P. O. Box 844
Northridge, California

Mr. J. W. Lower, Chief
Engineer-Inertial Components
Honeywell Aero Division
2600 Ridgway Road
Minneapolis, Minnesota

Mr. J. Levine
Ford Instrument Company
31-10 Thomson Avenue
Long Island City 1, New York

Mr. Adolf Egli
Ford Motor Company
Engineering and Research Staff
P. O. Box 2053
Dearborn, Michigan

Chairman
Research Committee on Lubrication
The American Society of Mechanical
 Engineers
United Engineering Center
345 East 47th Street
New York 17, New York   (2)

Dr. W. A. Gross
Ampex Corporation
401 Broadway
Redwood City, California   94063 (2)

Mr. Stanley L. Zedekar
Department 244-2, Building 71
Autonetics
P. O. Box 4181
Anaheim, California   92803

C. D. Flanigen
Director of Engineering
Lycoming Division, Avco Corporation
Stratford, Connecticut

Dr. Russell Dayton
Battelle Memorial Institute
505 King Avenue
Columbus 1, Ohio

Mr. Ralph H. Larson
Research Laboratories Division
Bendix Aviation Corporation
Southfield, Michigan

Walt Tucker
Nuclear Engineering Department
Brookhaven National Laboratory
Upton, Long Island, New York

Jet Propulsion Laboratory
California Institute of Technology
4800 Oak Grove Avenue
Pasadena, California
Attn: Library

Dr. F. Osterle
Mechanical Engineering Department
Carnegie Institute of Technology
Pittsburgh, Pennsylvania   15213

Professor M. C. Shaw, Head
Department of Mechanical Engineering
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania

Dr. John E. Mayer, Jr.
Applied Research Office
Research and Engineering Center
Ford Motor Company
P. O. Box 2053
Dearborn, Michigan

Professor D. D. Fuller
Laboratories for Research and
 Development
The Franklin Institute
Philadelphia, Pennsylvania   (3)

Mr. Jerry Glaser
Senior Project Engineer (Dept 37)
AiResearch Manufacturing Division
The Garrett Corporation
9851 S. Sepulveda Boulevard
Los Angeles, California

Library
General Atomic Division
General Dynamics Corporation
P. O. Box 608
San Diego, California   92112

Mr. G. R. Fox, Manager
Bearing and Lubricant Center
General Engineering Laboratory
General Electric Company
1 River Road
Schenectady, New York   (1)

Mr. C. C. Moore, Specialist
Advanced Bearing & Seal Technology
ARADO - AETD, H-55
General Electric Company
Flight Propulsion Division
Cincinnati 15, Ohio

Mr. E. Roland Maki
Mechanical Development Department
Research Laboratories
General Motors Corporation
General Motors Technical Center
12 Mile and Mound Roads
Warren, Michigan

Mr. Howard F. Traeder
Instruments Engineering Department
AC Spark Plug Division
General Motors Corporation
Milwaukee, Wisconsin   53201

Mr. Walter Carow
Kearfott Division
General Precision Incorporated
1150 McBride Avenue
Little Falls, New Jersey

W. David Craig, Jr.
Mechanical Design Section
Grumman Aircraft Engineering Corporation
Bethpage, Long Island, New York

Hydronautics, Incorporated
Pindell School Road
Howard County
Laurel, Maryland

Professor L. N. Tao
Illinois Institute of Technology
Chicago 16, Illinois

Mr. Stanley Abramovitz, Director
Industrial Tectonics, Inc.
Fluid Film Bearing Division
New South Road and Commercial Street
Hicksville, Long Island, New York

Dr. W. E. Langlois
International Business Machines
 Corporation
Research Laboratory
San Jose, California   (1)

The Johns Hopkins University
Applied Physics Laboratory
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Fenton L. Kennedy
       Document Library

Mr. B. A. Napier
Director of Engineering
Lear Siegler, Inc.
Power Equipment Division
P. O. Box 6719
Cleveland 1, Ohio

Professor A. Charnes
The Technological Institute
Northwestern University
Evanston, Illinois

Mr. E. L. Swainson, Chief of Research
Precision Products Department
Nortronics
A Division of Northrop Corporation
100 Morse Street
Norwood, Massachusetts

Northrop Norair
3901 W. Broadway
Hawthorne, California   90250
Attn: Tech Info, 3924-32

Professor P. R. Trumpler
Towne School of Civil and Mechanical
 Engineering
University of Pennsylvania
Philadelphia, Pennsylvania

Pratt & Whitney Aircraft
Division of UAC - CANEL
P. O. Box 611
Middletown, Connecticut
Attn: Librarian

Radio Corporation of America
Camden 2, New Jersey
Attn: Library, Building 10-2-5

Mr. Robert S. Siegler
Rocketdyne
Nucleonics Subdivision
6633 Canoga Avenue
Canoga Park, California   (1)

Dr. Ralph A. Burton
Southwest Research Institute
San Antonio, Texas

Mrs. Florence Turnbull
Engineering Librarian
Sperry Gyroscope Company
Great Neck, New York

W. G. Wing
Sperry Gyroscope Company
2T120
Great Neck, New York

Mr. R. C. Blaylock
Vice President - Technical Director
Ling-Temco-Vought, Inc.
P. O. Box 5003
Dallas, Texas

Dr. J. S. Ausman
Litton Systems, Inc.
5500 Canoga Avenue
Woodland Hills, California   (2)

Mr. Den Moore
Litton Systems
5500 Canoga Avenue
Woodland Hills, California   (2)

Lockheed Aircraft Corporation
Missiles and Space Division
Technical Information Center
3251 Hanover Street
Palo Alto, California

Massachusetts Institute of Technology
Instrumentation Laboratory
68 Albany Street
Cambridge, Massachusetts   02139
Attn: Library, ILl-109

Dr. R. W. Mann (Room 3-459A)
Engineering Projects Laboratory
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Professor Herbert H. Richardson
Room 3-461
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

McDonnell Aircraft Corporation
Post Office Box 516
St. Louis, Missouri   63166
Attn: Library, Dept. 218

Dr. Beno Sternlicht
Mechanical Technology Incorporated
968 Albany-Shaker Road
Latham, New York   (3)

Mr. Carl F. Graesser, Jr.
Director of Research
New Hampshire Ball Bearings, Inc.
Peterborough, New Hampshire

Mr. O. Decker
Thomson Ramo Wooldridge
TAPCO Group
New Devices Laboratories
7209 Platt Avenue
Cleveland 4, Ohio

Union Carbide Nuclear Company
Division of Union Carbide Corporation
P. O. Box P
Oak Ridge, Tennessee   37831
Attn: R. G. Jordan

Professor J. Modrey
Department of Mechanical Engineering
Purdue University
Lafayette, Indiana

Ralph F. DeAngelias, Technical Librarian
Norden Division of United Aircraft
 Corporation
Helen Street
Norwalk, Connecticut   06852

Unidynamics/St. Louis Division Library
Universal Match Corporation
472 Paul Avenue
St. Louis, Missouri   63135

Mr. J. M. Gruber, Ch. Engrg.
Waukesha Bearings Corporation
P. O. Box 798
Waukesha, Wisconsin

John Boyd
Westinghouse Research Laboratories
Churchill Boro
Pittsburgh, Pennsylvania   15235

Mr. H. Walter
Vice President - Research & Development
Worthington Corporation
Harrison, New Jersey

Dr. Calus G. Goetzel, D/52-30
Bldg. 201, Plant 2, Palo Alto
Lockheed Missiles & Space Company
P. O. Box 504
Sunnyvale, California

Mr. Philip J. Mullan
Staff Engineer - Advanced Tapes
IBM Data Systems Division
Development Laboratory
Box 390, Poughkeepsie, New York   12602

General Dynamics/Convair
P. O. Box 1128
San Diego, California   92112
Attn: Library & Information Services (128-00)

Science & Technology Division
Library of Congress
Washington, D. C.   20540

Admiraldy Compass Observatory
Ditton Park
Slough, Bucks, England
Attn: Mr. Henri J. Elwertowski

The University of Southampton
Department of Mechanical Engineering
Southampton, England
Attn: Dr. H. S. Grassam

Mr. R. A. Minard
Assistant Product Manager
Gas Bearing Technology Division
MPB Incorporated
Precision Products Division
Keene, New Hampshire

Mr. Edgar J. Gunter, Jr.
University of Virginia
School of Engineering and Applied Science
Charlottesville, Virginia   22903

Mr. L. Licht
Department of Mechanical Engineering
Columbia University
New York, New York

Mr. Sherril Hisaw
The LaFleur Corporation
16659 Gramercy Place
Torrance, California

Dr. Charles C.W. Ng
IIT Research Institute
10 West 35 Street
Chicago, Illinois   60616

Mr. Randy J. Sherman, Manager
Turbine Department
Technical Operations, Inc.
Radiation Products Division
441 Whisman Road
Mountain View, California

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*	2a. REPORT SECURITY CLASSIFICATION
Ampex Corporation 401 Broadway Redwood City, California	Unclassified
	2b. GROUP

3. REPORT TITLE

THE DYNAMIC BEHAVIOR OF A FOIL IN THE PRESENCE OF A LUBRICATING FILM

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Technical Report

5. AUTHOR(S) *(Last name, first name, initial)*

Eshel, A. and
Wildmann, M.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1966		

8a. CONTRACT OR GRANT NO. Contract No. Nonr-3815(00)(X) b. PROJECT NO. Task NR 062-297 c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) RR 66-29 9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. AVAILABILITY/LIMITATION NOTICES

Qualified requesters may obtain copies of this report from DDC.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
None	Department of the Navy Office of Naval Research Washington, D.C., 20360

13. ABSTRACT

Equations for the oscillations of a foil over a lubricating fluid film are derived and are simplified by a small parameter expansion. A few particular cases are discussed, and a linearized solution is obtained for the case of a massless, perfectly flexible foil moving at a speed U over an incompressible film. The solution reveals the phenomenon that small disturbances in the film thickness, as well as symmetrical large disturbances, propagate at a speed $U/2$.

DD FORM **1473**
1 JAN 64

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
FOILS						
LUBRICATING FILM						
EQUATIONS						
MATHEMATICAL ANALYSIS						
FLUID MECHANICS						
OSCILLATION						
MECHANICAL PROPERTIES						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through

_____."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through

_____."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through

_____."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, rules, and weights is optional.

AMPEX